

Stable Q–balls from extra dimensions

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Abstract

Given a bulk scalar field with sufficient self–interactions in a higher dimensional spacetime, it is shown that the continuous symmetries in four dimensions, induced by the topological structure of the compact manifold, naturally lead to formation of stable nontopological solitons of Q–ball type. The mass per unit charge inside the soliton is bounded by the size of the extra dimensions, and it is thus stable with respect to decaying into excited levels of all bulk fields, irrespective of their bulk masses. A familiar example is the Standard Model in the bulk, where the Kaluza–Klein levels of the Higgs boson form a stable Q–ball. These stable solitons are natural candidates for the dark matter in the Universe.

1 Introduction

In all theories of Kaluza–Klein (KK) type [1] one hypothesizes that the space–time is $4 + \delta$ dimensional, and it has already a partially compactified structure $\mathbf{M}^4 \times \mathbf{B}^\delta$ [2], instead of $\mathbf{M}^{4+\delta}$, where \mathbf{B}^δ is a compact δ –dimensional space. The symmetries of $\mathbf{M}^4 \times \mathbf{B}^\delta$ are the four–dimensional Poincare symmetries acting on \mathbf{M}^4 and some internal symmetry group generated by the topological structure of \mathbf{B}^δ . The symmetries of the internal manifold are observed as the symmetries of the four–dimensional lagrangian. For instance, $\mathbf{M}^4 \times \mathbf{S}^1$ generates a $U(1)$ symmetry in four–dimensions whereas $\mathbf{M}^4 \times \mathbf{S}^2$ realizes an $SU(2)$ symmetry [3]. Unlike the common wisdom in KK theories where one attempts to construct realistic gauge theories in four dimensions from a higher dimensional theory with an appropriate internal manifold [3], here we consider a bulk field theory in higher dimensions, and discuss possible effects of KK symmetries in the four–dimensional theory.

The extra dimensions could be spacelike or timelike. Compactification of the timelike coordinates cause probability and causality violation, and thus, phe-

nomenologically they should not exceed Plackian sizes [4] unless there is some mechanism localizing particles in the extra time coordinate [5]. On the other hand, within the present experimental precision [6], the compact spacelike dimensions could be as large as a mm with important phenomenological implications [7]. Given a gauge theory in the bulk of $\mathbf{M}^4 \times \mathbf{B}^\delta$ spacetime, the scalar sector of this theory must be such that the gauge group should be spontaneously broken down to the subgroup required by physics in four dimensions. This is because of the fact that the KK compactification cannot lead to symmetry breaking in lower dimensions. Hence, in what follows we consider a bulk scalar field theory describing a physical spinless meson.

In what follows we will show that, irrespective of the symmetries and particle spectra of the field theory in $\mathbf{M}^4 \times \mathbf{B}^\delta$ bulk, the continuous symmetries of the four-dimensional effective theory generated by the topological structure of \mathbf{B}^δ enable the formation of nontopological solitons of Q-ball type [8] if the scalar sector possesses necessary interactions. This particularly necessitates that the internal manifold should be compact (*e.g.* there is no orbifolding), and at least the scalar fields should admit a KK expansion (*e.g.* they are not strictly localized on a topological defect [9,10]).

2 Kaluza–Klein Q–Balls

For simplicity we consider a $(1 + 4)$ -dimensional spacetime with a partially compactified structure $\mathbf{M}^4 \times \mathbf{S}^1$ [2]. Although we assume a 1-sphere for the internal space, the results of the analysis will be general enough to cover higher dimensional compact spaces \mathbf{B}^δ . As usual the metric tensor $G_{AB}(x_\mu, y)$ can be decomposed in terms of graviton $g_{\alpha\beta}(x_\mu, y)$, graviphoton $A_\alpha(x_\mu, y)$ and graviscalar $\sigma(x_\mu, y)$ all living in the bulk [1,11]. However, all essential features of KK Q-balls can be understood by analyzing a self-interacting bulk scalar field theory in a flat $\mathbf{M}^4 \times \mathbf{S}^1$ background. Therefore, the KK excitations in the graviton sector, which are not essential for Q-ball formation, can be switched off by replacing the metric tensor with its vacuum expectation value [12]

$$\langle G_{AB}(x_\mu, y) \rangle = \begin{pmatrix} \eta_{\alpha\beta} & 0 \\ 0 & -1 \end{pmatrix}. \quad (1)$$

Consequently, we consider a bulk scalar field $\Phi(x_\mu, y)$ with the action

$$S_5 = \int d^4x dy \left\{ \partial_\mu \Phi \partial^\mu \Phi - (\partial_y \Phi)^2 - \overline{m}^2 \Phi^2 - V(\Phi) \right\} \quad (2)$$

where \overline{m} is a bare mass parameter, and $V(\Phi)$ contains trilinear and higher order self-interaction terms.

Letting R be the radius of \mathbf{S}^1 , the periodicity of $\Phi(x_\mu, y)$ in y allows for the Fourier decomposition

$$\Phi(x, y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) \frac{e^{iny/R}}{\sqrt{2\pi R}}. \quad (3)$$

where $\phi_{-n}(x) = \phi_n^*(x)$ due to the reality of $\Phi(x_\mu, y)$. Hence, the theory in \mathbf{M}^4 now consists of an infinite number of complex scalars $\{\phi_n\}$

$$S_4 = \int d^4x \sum_n \left[\partial_\mu \phi_n \partial^\mu \phi_{-n} - (\overline{m}^2 + m_n^2) \phi_n \phi_{-n} - V_n(\phi) \right] \quad (4)$$

where $m_n = |n|/R$, and $V_n(\phi)$ follows from $V(\Phi)$ after using (3) and integrating over y . For instance, a cubic interaction in $V(\Phi)$ contributes to $V_n(\phi)$ by $\sum_k \phi_n \phi_k \phi_{-(n+k)}$. For the model under consideration, one can restrict the KK summations in (4) to $[0, \infty]$ interval by using the reality of $\Phi(x_\mu, y)$. However, when there are several scalar (not necessarily real) fields the notation of (4) is appropriate.

As is clear from (4), irrespective of what type of interactions and what kind of fields participate in $\mathbf{M}^4 \times \mathbf{S}^1$ theory, the lagrangian in \mathbf{M}^4 possesses a global $U(1)$ invariance generated by translations along \mathbf{S}^1 :

$$U(1)_{KK} : \phi_n(x) \rightarrow \phi_n(x) e^{in\alpha}, \quad (5)$$

and all bulk fields in the theory (*e.g.* fermions and gauge bosons) have the same transformation property as well. In particular, both KK charge and KK mass of the zero mode ϕ_0 vanishes identically. In general, symmetry properties of ϕ_0 are identical to those of $\Phi(x, y)$. Consequently, in the present analysis, it remains as a real scalar field.

By convention $V_n(0) = 0$, and $\{\phi_n\} = 0$ represents the ground state of the theory in which $U(1)_{KK}$ is unbroken. In this symmetric vacuum there are infinite number of spinless mesons with masses $\sqrt{\overline{m}^2 + m_n^2}$. However, as was shown by Coleman [8], extended objects appear in the spectrum if the total energy is minimized for fixed $U(1)_{KK}$ charge

$$Q \equiv \sum_n \frac{n}{i} \int d^3\vec{x} \phi_{-n}(\vec{x}, t) \overleftrightarrow{\partial}_t \phi_n(\vec{x}, t). \quad (6)$$

It is convenient to introduce a Lagrange multiplier ω to incorporate the constraint $Q \neq 0$ to the energy functional

$$E_\omega = \omega Q + \int d^3\vec{x} \sum_n \left[\nabla\varphi_{-n}\nabla\varphi_n + M_n^2(\omega) \varphi_{-n}\varphi_n + V_n(\varphi) \right] \quad (7)$$

where $M_n^2(\omega) = \overline{m}^2 + (1 - \omega^2 R^2) m_n^2$. This expression for energy is to be minimized with respect to ω and φ_n , independently. That the energy functional is independent of time follows from the fact that the scalars ϕ_n rotate in the internal space with velocities proportional to their $U(1)_{KK}$ charge

$$\phi_n(\vec{x}, t) = \varphi_n(\vec{x}) e^{in\omega t} . \quad (8)$$

From (7) it is clear that in the fixed KK charge sector the n -th KK level acquires a tachyonic mass $m_n^2\omega^2 R^2$, which is what enables φ_n to develop a nonvanishing VEV in a domain of finite charge accumulation. To minimize (7) for fixed ω one has to solve an infinite number of coupled nonlinear differential equations [14] with necessarily spherically symmetric solutions [15]. Q-balls exist for small [16] (but large enough to allow for a semiclassical analysis) as well as large [8] Q values. In the large Q (thin-wall) limit the minimal energy configuration is well approximated by a sphere of radius R_Q , such that $\varphi_n = \overline{\varphi}_n \neq 0$ for $|\vec{x}| \leq R_Q$, and $\varphi_n \equiv 0$ for $|\vec{x}| > R_Q$ [8]. For this lump of matter to be stable with respect to decaying into a free scalar ϕ_n its total energy (\equiv mass M_Q) must be less than the total energy $N_n M_n$ of $N_n = Q/n$ scalars having total charge Q . Then the stability condition takes the form [17]

$$\left(\frac{M_Q}{Q} \right)_{min} < \left(\frac{M_n(0)}{n} \right)_{min} , \quad (9)$$

where n refers to that KK state for which the right-hand side is a minimum. In this equation the left hand-side is a function of the KK condensates $\overline{\varphi}_n$

$$\left(\frac{M_Q}{Q} \right)_{min} = \left(\frac{4\pi}{3} R_Q^3 \frac{2\omega}{Q} \sum_n \{ M_n^2(0) \overline{\varphi}_n \overline{\varphi}_{-n} + V_n(\overline{\varphi}) \} \right)^{1/2} , \quad (10)$$

and the right-hand side equals

$$\left(\frac{M_n(0)}{n} \right)_{min} = \frac{1}{R} \quad (11)$$

independent of whether $\Phi(x_\mu, y)$ has a finite bare mass or not. Here the discussion has been restricted to thin-wall (large Q) regime in which $M_Q \sim Q/R$.

In the thick-wall (small Q) limit M_Q/Q receives a smooth Q dependence [16]. If the potential $V_n(\phi)$ is a flat one then $M_Q \propto Q^{3/4}$ rather than Q , and the value of the condensate inside the Q-ball can exceed the energy density by many orders of magnitude [18].

The results obtained here can be generalized to more complicated scalar field theories such as the supersymmetric theories with [19] or without [13] CP violation, or possible extensions of the standard Higgs sector [20]. If the number of extra dimensions increase, say $\delta = 2$ with a nontrivial \mathbf{S}^2 topology, then the four-dimensional theory has an $SU(2)$ symmetry which admits Q-ball formation [21].

The stability condition (9) requires that the mass per unit charge inside the Q-ball is bounded by the inverse compactification radius. According to the present experimental accuracy [6] $10^{-4} \text{ eV} \lesssim R^{-1} \lesssim M_{Pl}$. Therefore, if the potential $V(\Phi)$ admits a stable Q-ball solution then its mass per unit charge is constrained solely by the gravity experiments.

The zero mode ϕ_0 , which has neither KK mass nor KK charge, has the same Lagrangian and the same symmetries as the bulk scalar $\Phi(x_\mu, y)$ apart from rescalings of the couplings. However, depending on the details of the potential $V(\Phi)$ it couples to higher KK levels. For instance, a cubic interaction of the form $\phi_0\phi_n\phi_{-n}$ can give rise to a KK Q-ball similar to baryon or lepton balls in supersymmetry [13]. Since the exit or entrance of ϕ_0 to Q-ball domain does not affect the total charge; it behaves as a self-interacting real scalar field for $|\vec{x}| \geq R_Q$, and becomes a component of the Q-matter for $|\vec{x}| < R_Q$.

The discussions above have concentrated on a bulk scalar field theory, in particular, the gravity sector has been ignored by adopting the vacuum configuration (1) for the metric tensor. However, a proper analysis must take into account the full structure of the metric tensor $G_{AB}(x_\mu, y)$. Once this is done the particle spectrum is widened by components of $G_{AB}(x_\mu, y)$, *i.e.*, the graviton $g_{\alpha\beta}(x_\mu, y)$, the graviphoton $A_\alpha(x_\mu, y)$, and the graviscalar $\sigma(x_\mu, y)$. The appearance of the graviphoton implies that the $U(1)_{KK}$ symmetry is now promoted to a local invariance. However, this does not invalidate the analysis above as the gauged $U(1)_{KK}$ also admits a Q-ball solution [22]. The function of the graviscalar appears in the dilaton field $\bar{\sigma} \equiv \ln\{\sigma/(12\sqrt{\pi G_N})\}$, which itself is a candidate for the bulk scalar $\Phi(x_\mu, y)$ necessary for Q-ball formation. However, this requires the introduction self-interaction terms for $\bar{\sigma}(x_\mu, y)$ not following from the decomposition of $G_{AB}(x_\mu, y)$. After the compactification, like any matter field, all components of $G_{AB}(x_\mu, y)$ possess a $U(1)_{KK}$ invariance. Since the stability bound in (9) depends only on the KK masses and charges of the bulk scalar (which could be $\bar{\sigma}(x_\mu, y)$ itself) practically all the results derived above remain valid.

3 Electroweak Kaluza–Klein Q–balls

In this section we investigate the formation and stability of the KK Q–balls within bulk electroweak theory [23]. Clearly the KK reduction cannot lead to symmetry breaking so that the standard model gauge symmetry must be already broken in $\mathbf{M}^4 \times \mathbf{S}^1$ bulk. Therefore, the bulk Higgs boson field $h(x_\mu, y)$ replaces the bulk scalar $\Phi(x_\mu, y)$, and its self–interaction Lagrangian provides the necessary potential

$$V(h) = \overline{m}^2 h^2 + \overline{\lambda} \overline{v} h^3 + \frac{1}{4} \overline{\lambda} h^4 \quad (12)$$

where $\overline{v}^2 = \overline{m}^2 / \overline{\lambda} \approx (246.2 \text{ GeV})^2 / (2\pi R)$. Since there exists a nonvanishing cubic coupling, there is necessarily a Q–ball solution [17,8,20]. As the bulk Higgs scalar $h(x_\mu, y)$ couples to all standard model fields so does the resulting KK Q–ball. This then raises the question of stability with respect to decaying into the standard model spectrum. However, once a stable Q–ball solution is formed for (12) it is then clear that the disassociation of the Q–matter to any field, irrespective of its bulk mass, is forbidden kinematically. This follows from the stability bound (9), which states that the mass per unit charge inside the Q–ball is less than two–particle threshold. To illustrate this point consider a bulk fermion $\Psi(x_\mu, y)$ which may have a mass m_ψ . If this mass is not generated via the spontaneous symmetry breaking then $\Psi(x_\mu, y)$ does not couple to $h(x_\mu, y)$, and thus, there is no question of stability. On the other hand, if m_ψ is proportional to \overline{v} then Q–ball would evaporate [24] by emitting $\overline{\psi}_n \psi_n$ pairs from the surface were not it for the stability condition (9). Indeed, n –th KK level has a mass $|n|/R$ so that kinematically the KK Q–ball cannot decay to charged fermion pairs according to (9). This absolute stability property is exclusive to KK Q–balls in which the zero modes are devoid of KK charge, and mass per unit charge inside the Q–ball is bounded by the inverse compactification radius. For comparison, one may recall the lepton balls of supersymmetry [13] which necessarily evaporate by emitting neutrinos [24] from the surface. Irrespective of whether $U(1)_{KK}$ is global [8] or local [22] there is a Q–ball solution, and therefore, inclusion of gravity sector does not affect the stability of the Higgs KK Q–balls.

The discussion above would be sufficient for having stable Higgs KK Q–balls were not it for the absence of chiral fermions. Indeed, the KK reduction produces equal numbers of left– and right–handed fermions so that the effective four–dimensional theory does not possess a definite chirality. In fact, to have chiral fermions in four dimensions it is necessary to perform an orbifolding of the extra dimension, and this, however, breaks the $U(1)_{KK}$ symmetry down to its Z_2 subgroup, leaving no continuous symmetry to develop Q–balls.

However, orbifolding is not the only way of inducing chiral fermions in \mathbf{M}^4 . Indeed, if the dynamical theory in \mathbf{M}^5 possesses multiple discrete degenerate vacua then there is always a massless chiral fermion [25] localized on the topological defect interpolating among different vacuum domains [9]. To be specific, let $\Theta(x_\mu, y)$ be a bulk scalar field in \mathbf{M}^5 with two degenerate vacua, then there arises a wall profile $\Theta_{wall}(y)$ which interpolates between the vacua situated at $y \rightarrow \pm\infty$. In the potential induced by $\Theta_{wall}(y)$, the zero mode fermion possesses a definite chirality and remains stuck to the wall [25,9] whereas the excited levels are all paired to have no preferred chirality [10].

The localization mechanism above is valid for \mathbf{M}^5 where none of the dimensions is compact. For spacetimes with a compact coordinate *e.g.* $\mathbf{M}^4 \times \mathbf{S}^1$, besides appropriate modifications in the underlying theory in order to allow for the formation of the walls, one must require $M_{wall} \gg R^{-1}$ where M_{wall} is the characteristic mass scale of the wall-former, $\Theta(x_\mu, y)$. The position of the wall along \mathbf{S}^1 may have a slight x_μ dependence; $y_0 = y_0(x_\mu)$ [26,27]. However, since the translational symmetry along \mathbf{S}^1 is spontaneously broken, this position is equivalent to the static solution $\Theta_{wall}(y)$ as $y_0(x_\mu)$ will be swallowed by the graviphoton $A_\mu(x)$ to acquire a mass. To have a nontrivial tilting of the wall with respect to \mathbf{S}^1 the graviphoton should be projected out [26]. For this purpose, it is convenient to impose a global $U(1)_I$ invariance on the system under which the wall-former transforms as $\Theta(x_\mu, y) \rightarrow e^{iq_I \alpha} \Theta(x_\mu, y)$. In addition to this, there is a local $U(1)_{KK}$ invariance defined in (5). These two symmetries are, however, spontaneously broken down to a global $\tilde{U}(1)_{KK} \equiv \sin \alpha U(1)_I + \cos \alpha U(1)_{KK}$ invariance once Θ_n develops a VEV for a non-vanishing n where $|n| \in (0, M_{wall} R)$ [26,27]. Any n in this interval can be chosen to be the vacuum configuration, as the mapping between the vacuum circle and \mathbf{S}^1 yields a topologically stable configuration. The remnant $\tilde{U}(1)_{KK}$ invariance, however, can generate a KK Q-ball if the potential of the wall-former has enough interactions. Practically, the most important change arises in the stability condition (9) where now mass per unit charge inside the Q-ball is bounded by $|\cos \alpha|/R$ instead of $1/R$. Therefore, the breaking direction in $U(1)_I \times U(1)_{KK}$ space, α , enters the game. All the standard model particles, which are the zero modes of the corresponding fields in $\mathbf{M}^4 \times \mathbf{S}^1$, are localized on the wall staying on \mathbf{S}^1 . The level splittings ($\sim M_{wall}$ [9,10]) of their excited modes are much more coarser than the KK splitting ($\sim R^{-1}$). Ignoring the former (which would be exact in the limit when the wall shrinks to a δ -function) the latter enables the formation of the KK Q-ball. In such a picture the wall-former plays two rôles: (i) it traps the massless quarks, leptons, Higgs, and gauge fields [9] whereby reproducing the four-dimensional world, and (ii) its KK modes with global $\tilde{U}(1)_{KK}$ symmetry form a Q-ball of the type studied above. Since the tilt angle α determines the $\tilde{U}(1)_{KK}$ charge of every field, the kinematic stability of the Q-matter with respect to decaying into other particles still persists.

4 Conclusion

In this work we have shown that a bulk scalar theory with sufficient interactions naturally accomodate nontopological solitons of Q-ball type after compactifying over the extra dimensions. The mass per unit charge inside the soliton is bounded by the inverse compactification radius; hence, it is absolutely stable with respect to decaying into the KK towers of all particles in the theory, including the gravity sector. A more realistic setting involving chiral fermions with the inclusion of the gravity sector arises approximately within the tilted wall scenario of [26,27].

The fact that the KK Q-balls are stable makes them a perfect candidate for the dark matter in the universe. Apart from electroweak [20] or GMSB [29] Q-balls, in general, baryon or lepton balls suffer from evaporation [24]. In supersymmetric theories, for instance, it is necessary to produce large enough baryon or lepton balls [18] to have them survive up to the present epoch [28].

It is clear that, formation of the KK Q-balls is effectively a decompactification process. This follows from the fact that, at least for the bulk scalar sector, all excited KK modes are imprisoned into a finite spatial domain which contains a fixed KK charge. The topology of the extra dimensions determines the symmetries of the four-dimensional effective theory, and minimization of the energy for fixed conserved charge maps all KK modes into a nontopological stable soliton. The ultimate answer to the existence or absence of the Q-balls (the KK Q-balls in particular) will come from the present and future experiments [30].

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